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VELOCITIES OF MANTLE LOVE AND RAYLEIGH WAVES OVER MULTIPLE PATHS

By M. Nafi Toksöz and Ari Ben-Menahem

ABSTRACT

Phase velocities of Love waves from five major earthquakes are measured over six great circle paths in the period range of 50 to 400 seconds. For two of the great circle paths the phase velocities of Rayleigh waves are also obtained. The digitized seismograph traces are Fourier analyzed, and the phase spectra are used in determining the phase velocities. Where the great circle paths are close, the phase velocities over these paths are found to be in very good agreement with each other indicating that the measured velocities are accurate and reliable. Phase velocities of Love waves over paths that lie far from each other are different, and this difference is consistent and much greater than the experimental error. From this it is concluded that there are lateral variations in the structure of the earth's mantle. One interpretation of this variation is that the mantle under the continents is different from that under the oceans, since the path with the highest phase velocities is almost completely oceanic. This interpretation, however, is not unique and variations under the oceans and continents are also possible.

Group velocities are computed from the phase velocities and are also directly measured from the seismograms. The group-velocity curve of Love waves has a plateau between periods of 100 and 300 seconds with a shallow minimum at about 290 seconds. The sources of error in both Fourier analysis and direct time domain methods of phase velocity measurement are discussed.

Introduction

The dispersive properties of surface waves are being used extensively in studying the structure of the earth's crust and mantle. The accuracy by which the structure can be determined is limited by the accuracy of the observed values of the phase and group velocities. At relatively short periods (less than 60–70 seconds), the local effects on the waves are such that only regional, rather than universal, dispersion curves are justified. At very long periods, free oscillation data have accurately fixed the phase velocity curves for both Love and Rayleigh waves. For the intermediate periods, there have been many measurements of phase and group velocities of Love and Rayleigh waves (Satô, 1958; Nafe and Brune, 1960; Brune, Ewing and Kuo, 1961; Brune, Benioff and Ewing, 1961; Ben-Menahem and Toksöz, 1962; Båth and Arroyo, 1962; Matumoto and Satô, 1962). Most of the observational data, however, show some scatter, and various measurements differ by as much as, and occasionally more than, one per cent of the measured value (Kovach and Anderson, 1962). Part of this variation may be due to the path differences and lateral variations in the structure. Before this question and the question of the best model for the earth's mantle can be resolved, it is essential that more precise and consistent surface wave dispersion data is obtained.

As a first step in this direction, we have measured phase velocities of Love waves, over complete great circle paths, from five major earthquakes: New Guinea (February 1, 1938), Assam (August 15, 1950), Kamchatka (November 4, 1952), Mongolia (December 4, 1957), and Alaska (July 10, 1958). For two of these, Assam and Mongolia, phase velocities of Rayleigh waves are also determined. The epicenter, origin time, and other pertinent information regarding these earthquakes are given in table 1. The great circle paths are through Pasadena, California with one exception: Wilkes, Antarctica is used in addition to Pasadena for the Alaska earthquake. Figure 1 shows the great circle paths through Pasadena and Wilkes.

TABLE 1

LIST OF EARTHQUAKES AND COORDINATES OF THE RECORDING STATIONS

Name	Date	Origin Time	Epicenter	
			Latitude	Longitude
New Guinea	Feb. 1, 1938	19:04:21	05°00′ S	131°30′ E
Assam	Aug. 15, 1950	14:09:29	28°24′ N	096°42′ E
Kamchatka	Nov. 4, 1952	16:58:20	52°42′ N	160°18′ E
Mongolia	Dec. 4, 1957	03:37:45	45°15′ N	099°24′ E
Alaska	July 10, 1958	06:15:54	58°18′ N	136°54′ W
Stations	Pasadena, California		34°08′54″ N	118°10′18″ W
	Wilkes, Antarctica		66°35′ S	110°35′ E

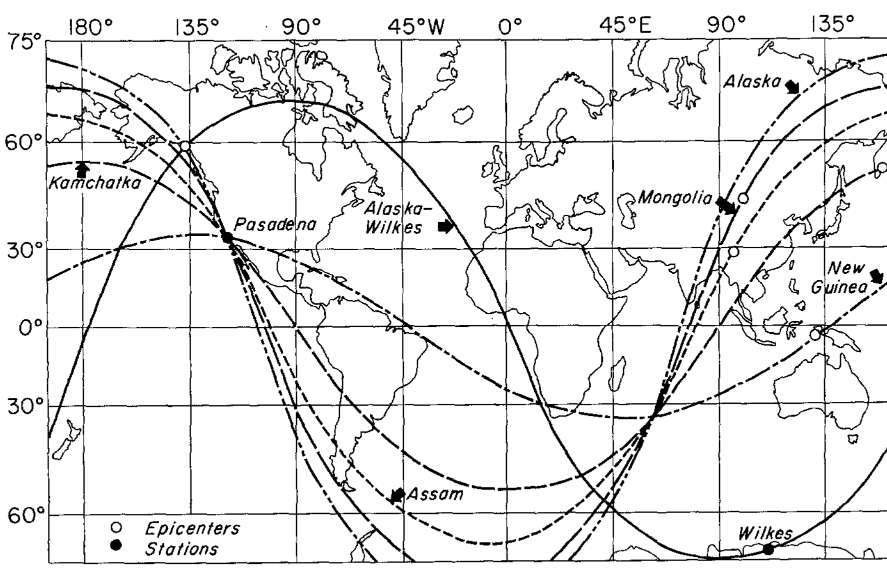

FIG. 1. Great circle paths.

With the exception of New Guinea, and Alaska-Wilkes, all paths are fairly close to one another, and one would expect the measured phase velocities to be approximately the same regardless of the lateral variation in mantle structure.

In all cases, the phase velocities were determined by the Fourier analysis method where the difference between the phase spectra of two successive passages of an odd or even order wave is used to determine the time delay for each phase. This method has become well-known since it was introduced by Valle (1949) and used by Satô (1958). In this paper a brief description of the method is given prior to its application to individual cases. The phase velocities from each one of the five earthquakes are then determined, and the group velocities are derived from the phase velocities. The second half of the paper is devoted to a comparison of the Fourier analysis method of phase velocity determination with direct measurement from the record. The sources of error in the Fourier analysis method are described and an approximate error bound is given.

PHASE VELOCITY DETERMINATION USING FOURIER PHASE SPECTRA

In this method, Fourier phase spectra are used for determining the phase delay of a given frequency between two stations. Let us take two recording stations over the same great circle path, with distances Δ_1 and Δ_2 from the epicenter. Assume that a Love or Rayleigh wave train is recorded at both stations. The desired wave train can be digitized and Fourier analyzed. Let t_1 and t_2 be the travel time from the source to the beginning of each Fourier window and $\phi_1(\omega)$ and $\phi_2(\omega)$ be the corresponding Fourier phase spectra. Then, the phase velocity C is given by

$$C(T) = \frac{\Delta_2 - \Delta_1}{t_2 - t_1 + T[(\phi_2 - \phi_1) + N]} \tag{1}$$

where T is the period and N is an integer. The need for N arises from the fact that the trigonometric functions are multi-valued, and one cannot determine uniquely the initial integer of ϕ_2 relative to ϕ_1. Once N is fixed, it remains unchanged over the whole spectrum. The changes in phase will normally exceed one circle, and every time ϕ goes through zero it is incremented by one circle.

When the phase velocities are determined from a single station, two successive passages of the same wave in the same direction such as G_1 and G_3, G_2–G_4, or R_3–R_5 are used for determining the phase velocity. In this case formula (1) becomes

$$C(T) = \frac{\Delta_0}{\delta t + T(\delta\phi + N + \frac{1}{2})} \tag{2}$$

where Δ_0 = length of great circle, $\delta t = t_{n+2} - t_n$, $\delta\phi = \phi_{n+2} - \phi_n$, and the $+\frac{1}{2}$ circle phase shift is due to two extra polar passages, with a $\pi/2$ phase shift per polar passage (Brune, Nafe, and Alsop, 1961).

The use of pairs of odd or even order surface waves, rather than odd-even combinations, in velocity measurement, is necessary to avoid the possibility of introducing an error due to the unknown character of the source. The source may contribute in a different way to waves leaving it in opposite directions. An error of this kind arises, for example, from the finiteness of source (Ben-Menahem, 1961). The differential phase between displacements of two opposite-going surface waves is given by:

$$D\phi = \frac{\phi_{n+1} - \phi_n}{2\pi} = \frac{f}{C}(\Delta_0 - 2\Delta_1 + b\cos\theta_0) + M + \frac{1}{4} \quad n = 1, 3, 5 \cdots \tag{3}$$

and a similar formula for even values of n. Δ_1 is the distance from the source to the station, b = fault length, θ_0 = angle between the fault trace and the great circle path. The term involving $b \cos\theta_0$ is due to the finiteness of the source, and a fault of 100 km can perturb the phase as much as 0.2 circles at the period of 100 seconds. This is a considerable error in the phase, and it cannot be neglected.

Phase Velocities

Phase velocities are determined for six great circle paths for Love waves; for two of these paths we also have the phase velocities of Rayleigh waves. The pertinent

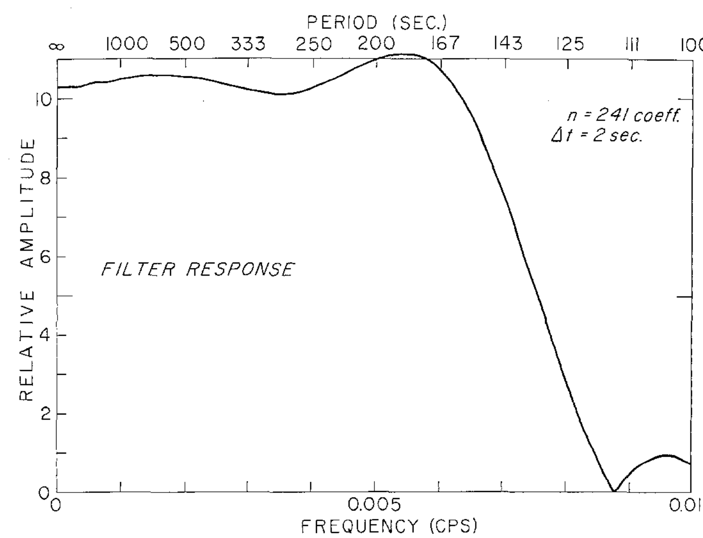

Fig. 2. Typical low-pass digital filter response.

waves were digitized with the sampling interval of two seconds (Mongolia G waves at 1 second intervals) and the digital data were then processed on an IBM 7090 computer where it was detrended, filtered, and plotted. In the case of Love waves it is difficult to determine the beginning and the end of the pulse by inspecting the unfiltered seismogram. Filtering process clears the wave form of excessive interference and facilitates choosing the window. In the case of Rayleigh waves a group velocity window can be used in determining the onset and the end of the pulse. The filtering consists of low-pass filtering with a cut-off frequency so chosen as to minimize interference and noise and also to reject the frequency bands that do not have sufficient power in both pulses. A typical filter response is shown in figure 2. Table 2 lists the onset and end times of the filtered pulses used in the analyses, the cut-off frequency of the low-pass filter used, epicentral distance to station, and the length of the great circle for each earthquake. The filtered pulses were Fourier analyzed and the phases were then used in equation 2 to compute the phase velocities.

Rayleigh Waves

(a) *Mongolia Earthquake.* The phase velocities of Rayleigh waves were computed and published earlier (Ben-Menahem and Toksöz, 1962). These same values are listed in table 3 after slight correction for the length of the great circle path.

(b) *Assam Earthquake.* The phase velocities are computed from R_3 and R_5, for great circle path through Pasadena. The amplitude and phase spectra are shown in figure 3 and the phase velocities are tabulated in table 3. The original traces were exhibited by Ewing and Press (1954).

TABLE 2

LIST OF PHASES, FILTER CHARACTERISTICS, AND EPICENTRAL DISTANCES

Earthquake	Station	Phase	Time of the Pulse		Filter Cut-off Period (second)	Epicentral Distance	Length of Great Circle Path
			Onset	End			
New Guinea	Pasadena	G_1	19:47:40	19:51:40	55	12194	40054
	Pasadena	G_2	20:48:04	20:53:02	55		
	Pasadena	G_3	22:17:52	22:24:40	55		
	Pasadena	G_4	23:16:40	23:23:48	55		
Assam	Pasadena	R_3	17:42:53	18:19:53	90	12174	40022
	Pasadena	R_5	20:23:53	21:30:51	90		
	Pasadena	G_1	14:57:23	15:01:23	67		
	Pasadena	G_3	17:27:47	17:36:33	67		
Kamchatka	Pasadena	G_2	19:05:06	19:10:26	100	6539	40032
	Pasadena	G_4	21:35:06	21:46:00	100		
Mongolia	Pasadena	G_1	04:16:15	04:21:50	80	10434	40018
	Pasadena	G_3	06:46:15	06:55:45	80		
Alaska	Pasadena	G_2	08:31:16	08:42:26	125	3025	40016
	Pasadena	G_4	10:59:52	11:15:18	125		
Alaska	Wilkes	G_2	07:38:00	07:48:12	125	16583	40018
	Wilkes	G_4	10:04:52	10:20:50	125		

Love Waves

(a) *Mongolia Earthquake.* The unfiltered Pasadena strain seismogram showing G_1 and G_3 are given in figure 4, and the spectra are given in figure 5. Phase velocities are listed in table 4.

(b) *Assam Earthquake.* The Pasadena E-W strain recordings of G_1 and G_3 are used for phase velocity measurement. Seismogram traces are shown in figure 6, and the spectra in figure 7.

(c) *Alaska Earthquake.* The G_2 and G_4 phases recorded by E-W component on the Press-Ewing seismograph system were used in determining the phase velocities over a great circle path through Pasadena. The re-traced unfiltered and filtered

TABLE 3

PHASE VELOCITIES OF RAYLEIGH WAVES

Frequency (cps)	Period (sec.)	Phase Velocity (km/sec)	
		Mongolia, R_5–R_3	Assam, R_5–R_3
.0028	357.14	5.627	—
.0029	344.83	5.554	—
.0030	333.33	5.485	5.509
.0031	322.58	5.415	5.428
.0032	312.50	5.358	5.364
.0033	303.03	5.301	5.312
.0034	294.12	5.229	5.244
.0035	285.71	5.171	5.179
.0036	277.78	5.118	5.121
.0037	270.27	5.060	5.067
.0038	263.16	5.003	5.004
.0039	256.41	4.957	4.957
.0040	250.00	4.913	4.919
.0041	243.90	4.862	4.867
.0042	238.09	4.817	4.818
.0043	232.56	4.781	4.780
.0044	227.27	4.744	4.743
.0045	222.22	4.710	4.713
.0046	217.39	4.682	4.680
.0047	212.77	4.651	4.648
.0048	208.33	4.612	4.613
.0049	204.08	4.588	4.595
.0050	200.00	4.568	4.566
.0052	192.31	4.523	4.524
.0054	185.19	4.479	4.483
.0056	178.57	4.434	4.436
.0058	172.41	4.398	4.403
.0060	166.67	4.370	4.381
.0062	161.29	4.349	4.339
.0064	156.25	4.316	4.319
.0066	151.52	4.288	4.291
.0068	147.06	4.279	4.271
.0070	142.86	4.268	4.253
.0072	138.89	4.255	4.243
.0074	135.14	4.247	4.232
.0076	131.58	4.234	—
.0078	128.21	4.217	4.227
.0080	125.00	4.203	4.224

pulses are shown in figure 8. The spectra are shown in figure 9. Phase velocities were also computed from G_2–G_4 combination of the recordings at Wilkes. The original seismogram and the filtered traces are shown in figure 10. The spectra are given in figure 11. Although the recordings were excellent at this station, the drum speed was not uniform causing the results to be somewhat unreliable.

(d) *Kamchatka Earthquake.* G_2 and G_4 phases from the Pasadena recordings of this earthquake were used for the determination of the phase velocity. The original

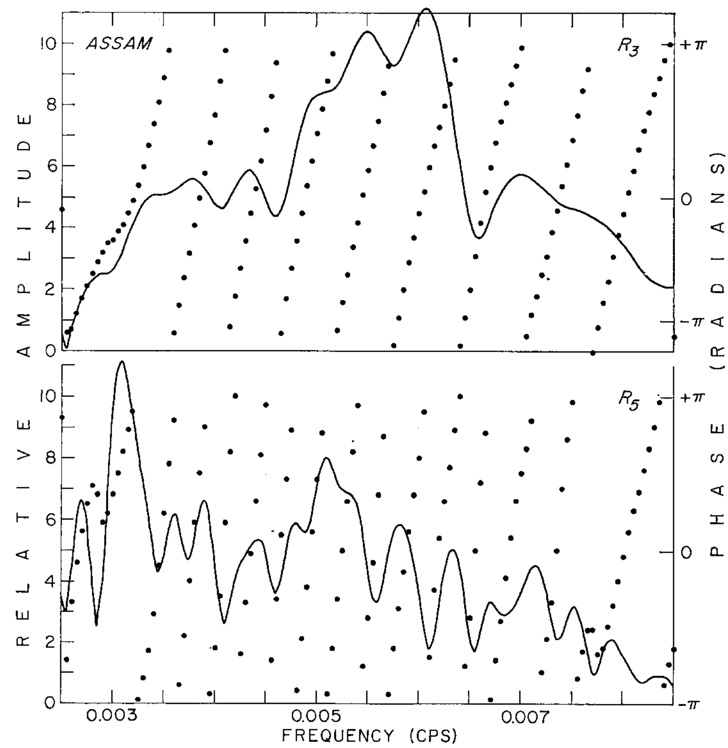

FIG. 3. Amplitude and phase spectra of R_3 and R_5 of the Assam earthquake

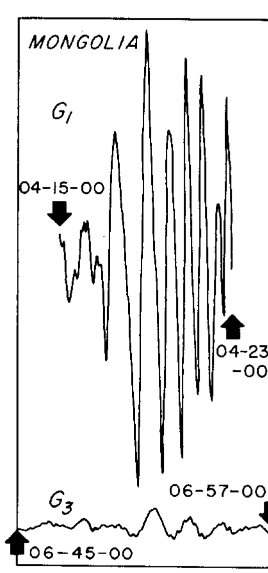

FIG. 4. G_1 and G_3 traces from the Mongolia earthquake recorded by Pasadena E-W strain seismograph.

seismogram was exhibited by Satô (1958). The filtered traces, however, are given in figure 12, and the corresponding spectra in figure 13.

(e) *New Guinea Earthquake.* G_1–G_3 and G_2–G_4 combinations were used to obtain two sets of phase velocities for the same path. The original Pasadena strain seismogram along with spectra of these phases had been given by Satô (1958). Since ours is a completely independent analysis we are showing the filtered traces and spectra in figure 14 and figure 15, respectively. The phase velocities are listed in table 4.

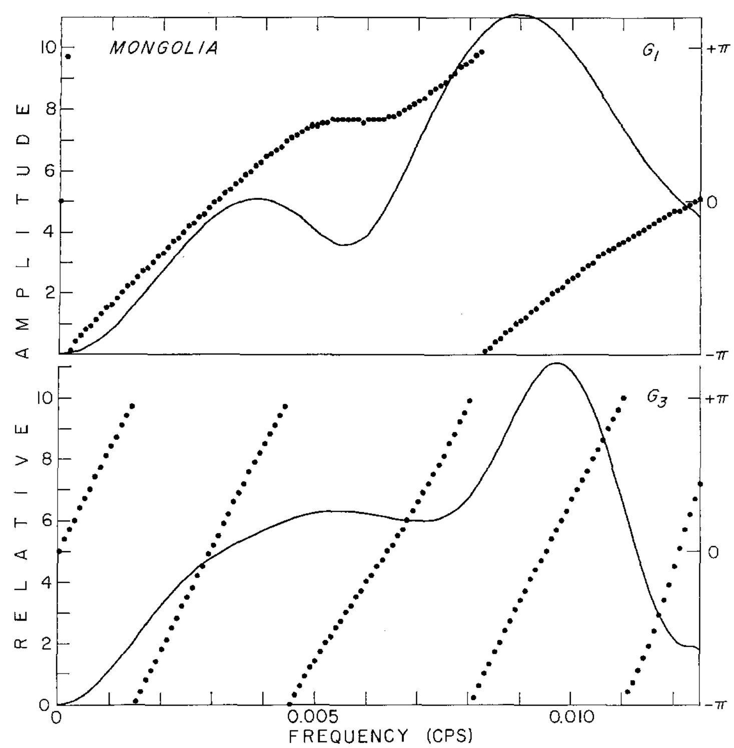

FIG. 5. Amplitude and phase spectra of Mongolia G_1 and G_3 .

For the sake of an easy comparison, the phase velocities of Rayleigh waves are plotted in figure 16, and those of Love waves in figure 17.

ANALYTIC EXPRESSIONS FOR PHASE AND GROUP VELOCITIES

The group velocities can be computed from the phase velocities, using the expression

$$U = \frac{d\omega}{dk} = \frac{C}{1 + \dfrac{T}{C}\dfrac{dC}{dT}} \tag{4}$$

where U = group velocity, $k = 2\pi/\lambda$ = wave number. The only difficulty in this computation arises from the differentiation of the phase velocity curve. This could

TABLE 4

PHASE VELOCITIES OF LOVE WAVES

Frequency (cps)	Period (sec.)	Phase Velocities (km/sec)						
		Mongolia G_3–G_1	Assam G_3–G_1	Alaska-Pasadena G_4–G_2	Kamchatka G_4–G_2	Alaska-Wilkes G_4–G_2	New Guinea G_3–G_1	New Guinea G_4–G_2
.0026	384.61	5.506		5.509				
.0027	370.37	5.452						
.0028	357.14	5.403		5.414				
.0029	344.83	5.358						
.0030	333.33	5.317		5.331		5.324		
.0031	322.58	5.279						
.0032	312.50	5.244		5.258	5.218	5.274		
.0033	303.03	5.211	5.180					
.0034	294.12	5.181	5.155	5.193	5.167	5.221		
.0035	285.71	5.153	5.131					
.0036	277.78	5.126	5.109	5.137	5.119	5.171		
.0037	270.27	5.102	5.089					5.134
.0038	263.16	5.078	5.070	5.089	5.076	5.123		5.113
.0039	256.41	5.057	5.052					5.093
.0040	250.00	5.036	5.034	5.047	5.037	5.078		5.074
.0041	243.90	5.017	5.018					5.055
.0042	238.09	4.998	5.002	5.010	5.002	5.037		5.038
.0043	232.56	4.981	4.987					5.022
.0044	227.27	4.965	4.973	4.977	4.971	5.001		5.007
.0045	222.22	4.949	4.958					4.993
.0046	217.39	4.934	4.945	4.948	4.942	4.968		4.973
.0047	212.77	4.920	4.932					4.966
.0048	208.33	4.907	4.919	4.921	4.916	4.940		4.953
.0049	204.08	4.895	4.907					4.942
.0050	200.00	4.882	4.894	4.896	4.891	4.914	4.942	4.930
.0052	192.31	4.860	4.872	4.874	4.869	4.891	4.918	4.909
.0054	185.19	4.840	4.851	4.853	4.848	4.870	4.897	4.889
.0056	178.57	4.821	4.832	4.834	4.829	4.851	4.877	4.871
.0058	172.41	4.804	4.814	4.816	4.810	4.834	4.859	4.854
.0060	166.67	4.789	4.797	4.800	4.791	4.818	4.842	4.839
.0062	161.29	4.774	4.780	4.785	4.772	4.804	4.826	4.824
.0064	156.25	4.760	4.765	4.771	4.754	4.792	4.812	4.811
.0066	151.52	4.748	4.750	4.758	4.737	4.781	4.801	4.798
.0068	147.06	4.736	4.736	4.746	4.723	4.770	4.787	4.786
.0070	142.86	4.724	4.723	4.734	4.711	4.761	4.775	4.775
.0072	138.89	4.713	4.710	4.724	4.701	4.752	4.765	4.765
.0074	135.14	4.703	4.698	4.714	4.692	4.744	4.755	4.755
.0076	131.58	4.693	4.686	4.704	4.685	4.736	4.745	4.746
.0078	128.21	4.683	4.676	4.695	4.680	4.729	4.736	4.737
.0080	125.00	4.674	4.666	4.686	4.674	4.722	4.728	4.729
.0082	121.95	4.665	4.657					
.0084	119.05	4.657	4.648				4.713	4.714
.0086	116.28	4.648	4.640					
.0088	113.64	4.642	4.633				4.699	4.701
.0090	111.11	4.635	4.626					
.0092	108.70	4.628	4.620				4.687	4.689
.0094	106.38	4.621	4.614					
.0096	104.17	4.615	4.609				4.677	4.678

TABLE 4 (*Cont.*)

Frequency (cps)	Period (sec.)	Phase Velocities (km/sec)						
		Mongolia G_3–G_1	Assam G_3–G_1	Alaska-Pasadena G_4–G_2	Kamchatka G_4–G_2	Alaska-Wilkes G_4–G_2	New Guinea	
							G_3–G_1	G_4–G_2
.0098	102.04	4.609	4.603					
.0100	100.00	4.603	4.598				4.667	4.668
.0102	98.04	4.597	4.593					
.0104	96.15	4.592	4.589				4.659	4.659
.0106	94.34	4.587	4.585					
.0108	92.59	4.582	4.580				4.651	4.651
.0110	90.91	4.577	4.576					
.0112	89.29	4.572	4.572				4.643	4.643
.0114	87.72	4.568	4.568					
.0116	86.21	4.564	4.565				4.635	4.637
.0118	84.75	4.559	4.561					
.0120	83.33	4.555	4.557				4.622	
.0124	80.64		4.550				4.609	
.0128	78.12		4.543					
.0132	75.76						4.595	
.0140	71.43						4.585	
.0148	67.57						4.576	
.0156	64.10						4.568	
.0164	60.98						4.560	
.0172	58.14						4.553	
.0180	55.56						4.546	

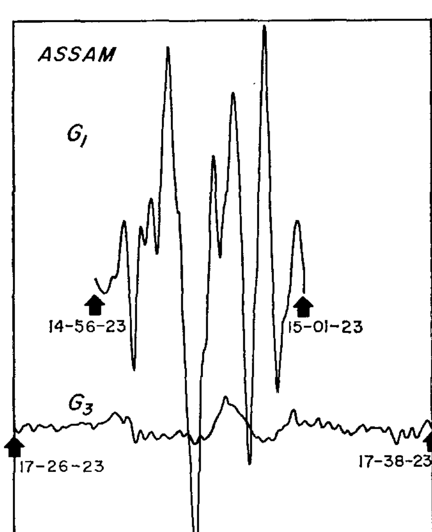

FIG. 6. Unfiltered traces of Pasadena strain recordings of G_1 and G_3 from the Assam earthquake.

be accomplished by either a direct numerical differentiation or representing the phase velocity curve by an analytic function and computing the derivatives of this function. We chose to fit a polynomial of the form $P_n(T) = \sum_{i=0}^{n} a_i X(T)$ to the data where the coefficients were determined by the method of least squares. Different polynomials of order $n = 4$ to $n = 9$ were used. The lower order polynomials miss the finer variations in the data, whereas the higher order polynomials follow any scattering that may be present in the original data resulting in undesired oscillations. The group velocities shown in figures 16 and 17 are the average values

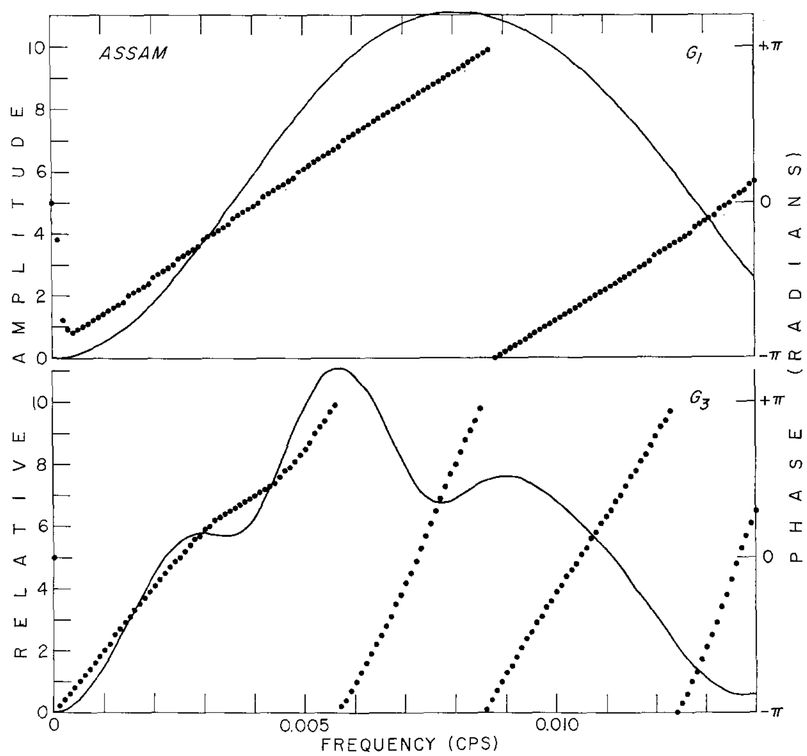

FIG. 7. Spectra of G_1 and G_3 from the Assam earthquake.

derived from two or more different order polynomials. It is important to mention that differentiation magnifies greatly the error that may be present in the phase velocity data, and the group velocities computed by this method are much less reliable than the phase velocities. In spite of this, these group velocities agree reasonably well with the velocities computed directly from the filtered records. The value of the method of computing the group velocity from the phase velocities, is greater for Love waves than for Rayleigh waves. The group velocity curve of Love waves is nearly flat from $T = 100$ to $T = 300$ seconds. The wave disperses very little and tends to preserve its initial shape. Unless one performs an extensive amount of narrow band filtering, it is very difficult to compute group velocities for different periods. This is one of the reasons for the greater scatter in the time domain meas-

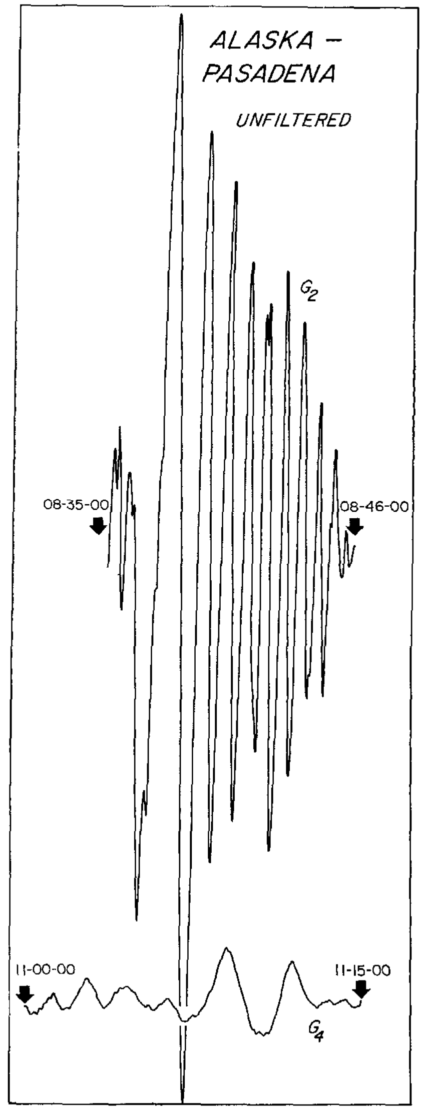

FIG. 8a

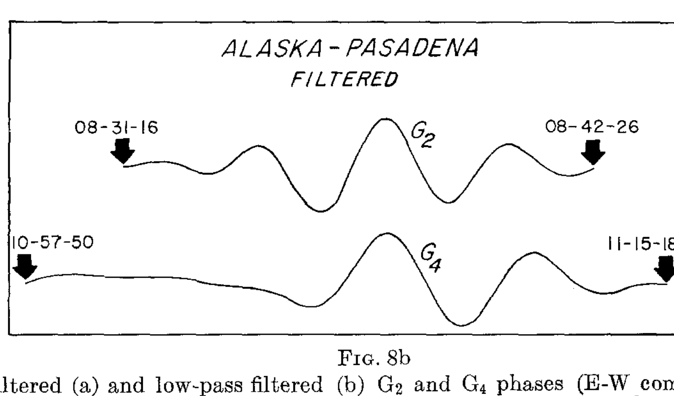

FIG. 8b

FIG. 8. Unfiltered (a) and low-pass filtered (b) G_2 and G_4 phases (E-W component) from the Alaska earthquake recorded at Pasadena by Press-Ewing seismograph system. (Amplitude scales are not uniform.)

urements of phase and group velocities of Love waves in this particular period range (see Brune, Benioff and Ewing, 1961, figure 7).

For more general use, simple functions can be fit to phase velocity data. For Rayleigh waves, one may write (Ben-Menahem and Toksöz, 1962)

$$C(T) = 3.85 + 0.0046T - 0.25 \sin (0.01T + 0.28) \qquad 100 < T < 500 \tag{5}$$

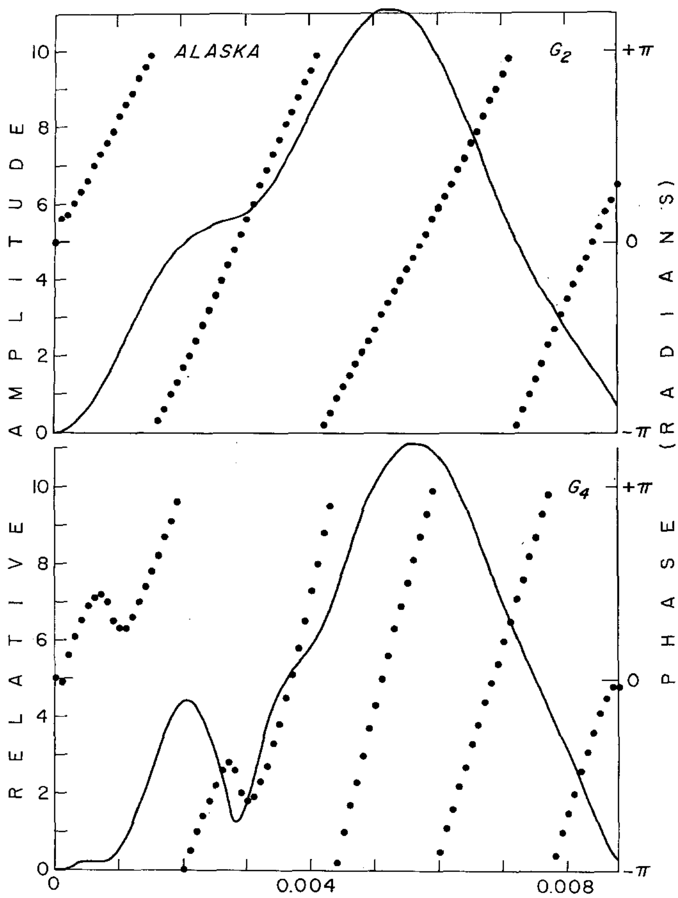

FIG. 9. Amplitude and phase spectra of Pasadena recodings of G_2 and G_4 from the Alaska earthquake.

which is a good approximation. Also, the group velocities obtained from the above expression using eq. (4) agree with the measured values. For the Love waves one may utilize the property of the almost-constancy of the group velocity in the plateau of the dispersion curve to derive an expression for the phase velocity (Satô, 1958). Differentiating the group velocity, and setting $dU/dT = 0$, one obtains the differential equation

$$2\left(\frac{dC}{dT}\right)^2 = C\frac{d^2C}{dT^2} \tag{6}$$

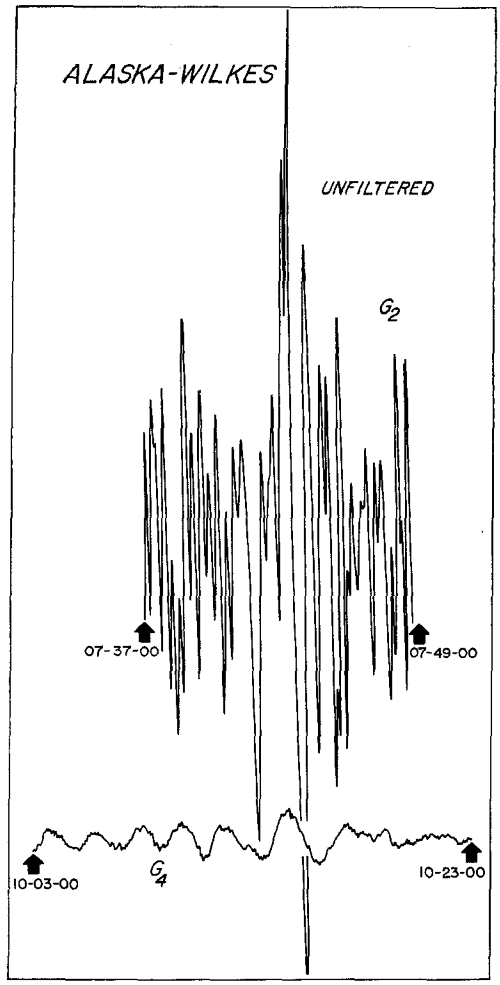

FIG. 10a

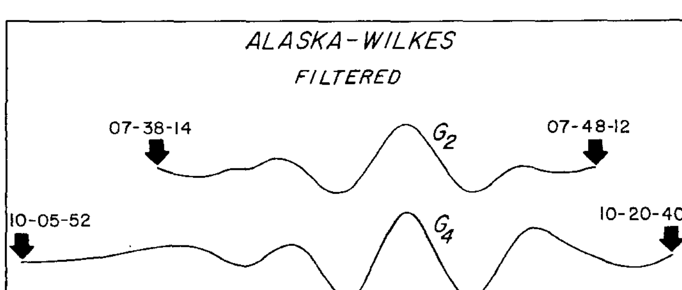

FIG. 10b

FIG. 10. Unfiltered (a) and low-pass filtered (b) G_2 and G_4 phases (N-S component) from the Alaska earthquake recorded at Wilkes by Press-Ewing seismograph. (Arbitrary amplitude scales.)

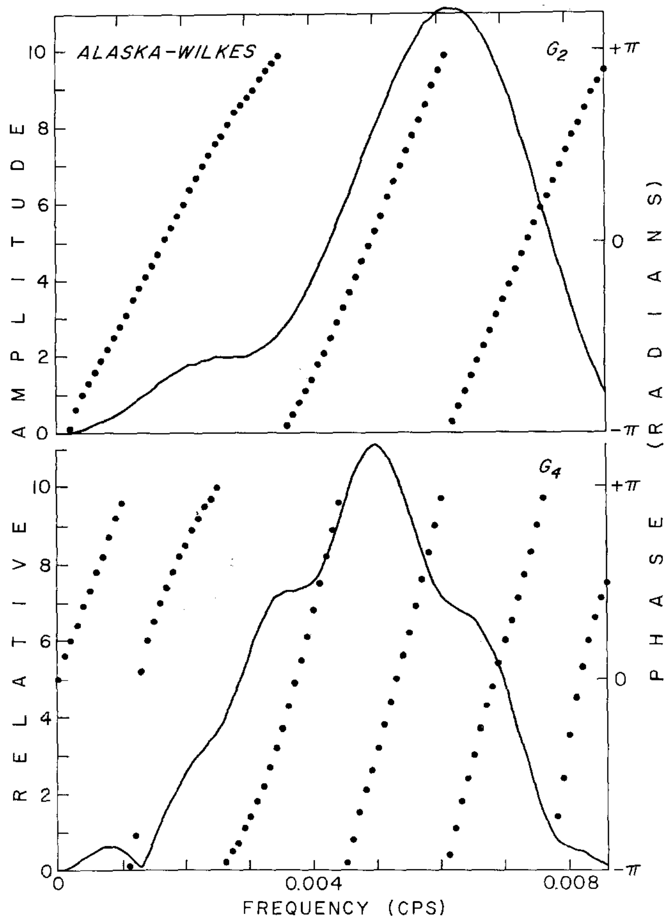

FIG. 11. Spectra of G_2 and G_4 from the Alaska earthquake recorded at Wilkes.

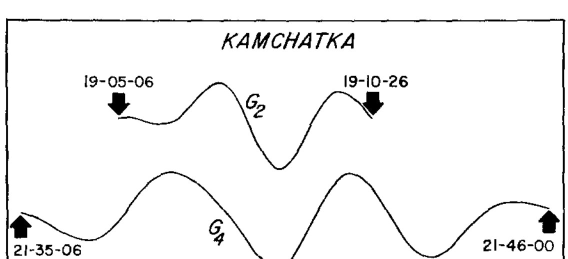

FIG. 12. Filtered traces of Pasadena N-S strain recordings of G_2 and G_4 from the Kamchatka earthquake. (Arbitrary amplitude scales.)

A solution to this equation is,

$$C = \frac{U_0}{1 - \alpha T} \tag{7}$$

where U_0 is the constant value of the group velocity. It should be noted that expression (6) is approximate, since the group velocity in the plateau is not a true

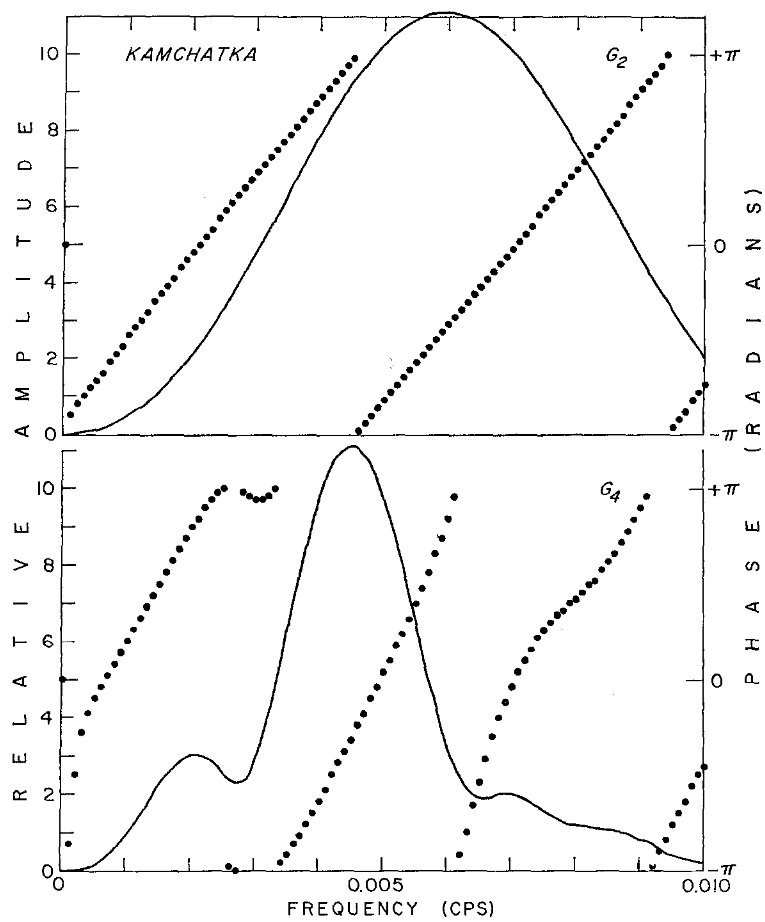

FIG. 13. Spectra of G_2 and G_4 from Kamchatka.

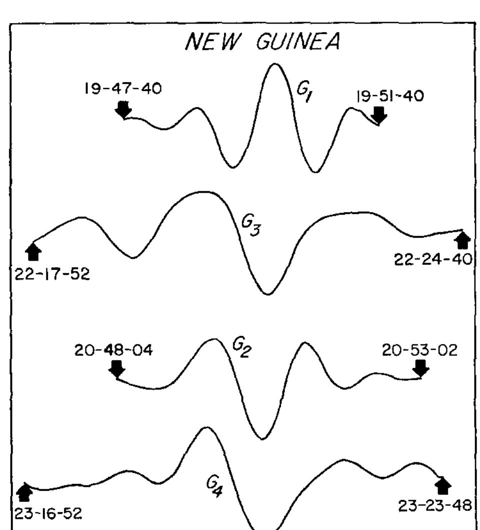

FIG. 14. Filtered traces of Pasadena N-S strain recordings of G_1, G_2, G_3, and G_4 traces from the New Guinea earthquake. (Arbitrary amplitude scales.)

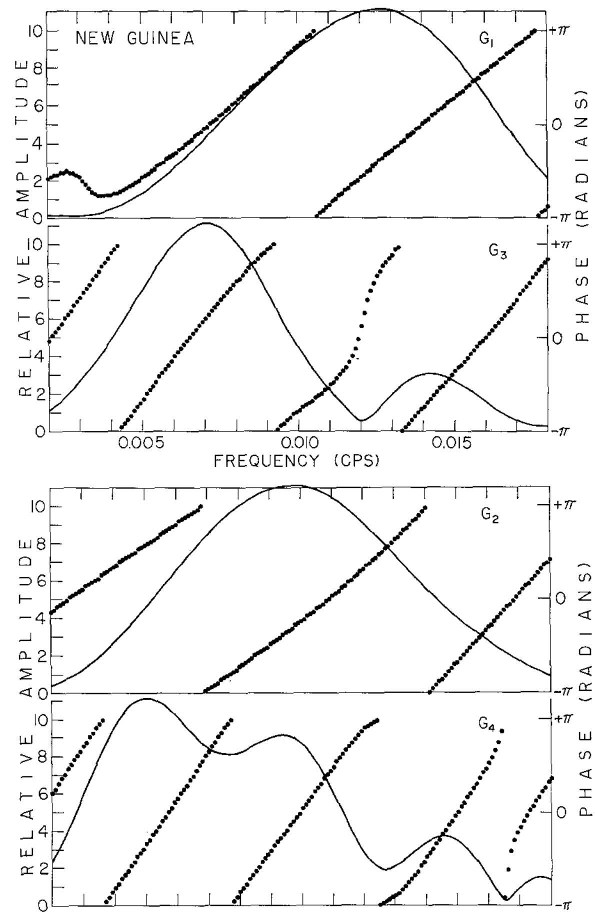

FIG. 15. Amplitude and phase spectra of G_1, G_2, G_3, and G_4 from New Guinea.

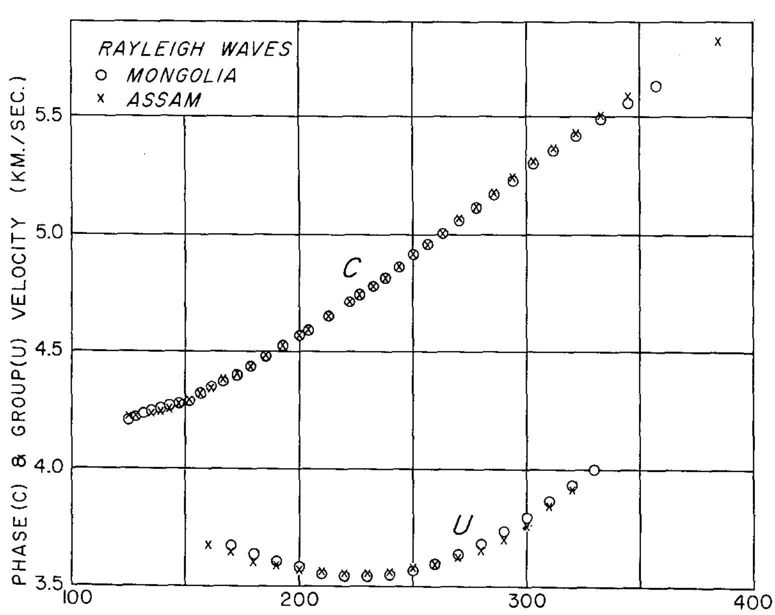

FIG. 16. Phase and group velocities of Rayleigh waves for Mongolia-Pasadena and Assan Pasadena great circle paths. Group velocities are derived from phase velocities.

constant but varies slowly with the period. The values $U_0 = 4.37$ and $\alpha = 5.35 \times 10^{-4}$ give a reasonably good fit to the observed phase velocities (New Guinea excepted) between $T = 100$ and $T = 300$ seconds. This indicates that the group velocity curve is nearly constant over that period range.

Discussion on the Accuracy of the Results

The phase velocities listed in tables 2 and 3 are probably within 0.5 per cent of the correct value. Since the absolute values are not known, reproducibility and the

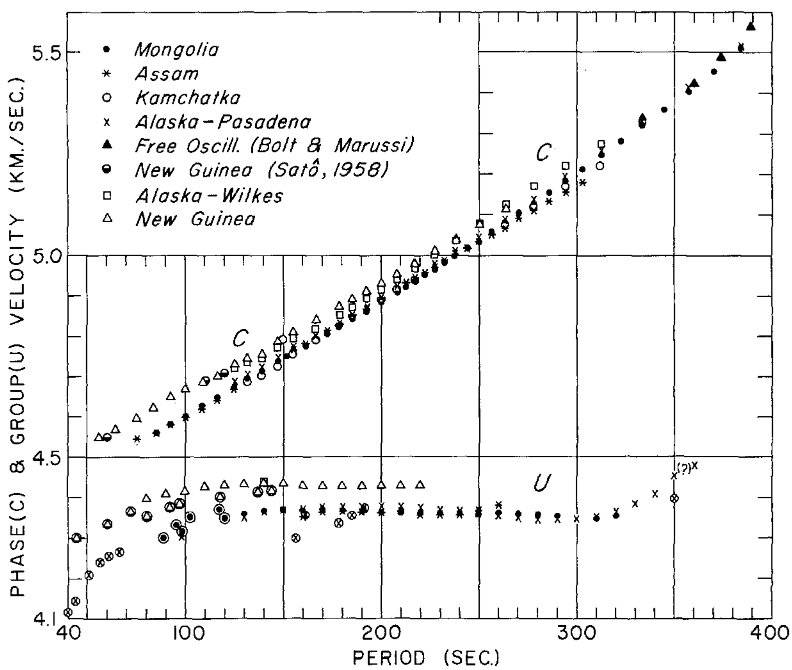

FIG. 17. Phase and group velocities of Love waves over several great circle paths. The group velocities with circles around the symbol are measured directly from the seismogram. Others derived from the phase velocities.

agreement between different measurements are the only basis for the judgment of accuracy. For the Mongolian and Assam earthquakes phase velocities agree with less than .02 km/sec discrepancy, over a wide frequency range, for both Love waves and Rayleigh waves. Phase velocities of Love waves from the Alaska earthquake (through Pasadena) agree with those of Assam and Mongolia, while Kamchatka yields slightly lower phase velocities. The New Guinea-Pasadena path has the highest values of phase velocities among all the paths. As to the reliability of these results, we can point out that the agreement between G_1–G_3 and G_2–G_4 combinations is excellent, and these results are also in accord with Satô's measurements as given by Brune, Benioff, and Ewing (1961). It should be noted here that the New Guinea-Pasadena great circle path is quite different from the other paths, being almost entirely oceanic. Alaska-Wilkes great circle phase velocities are between those of New Guinea and Mongolia. The slope of the phase velocity curve, however, is greater than those of others shown in figure 17. This discrepancy may be due to the larger error in Wilkes phase velocities compared to the others. It is also possible that the Alaska-Wilkes path represents a considerably different structure than the other paths.

The group velocities for different paths vary in a manner similar to the variation of phase velocities. For close paths (Mongolia, Alaska, Assam), the group velocities are in agreement. The New Guinea-Pasadena path, on the other hand, has considerably higher group velocities compared to others. In computing the group velocities from the seismograms, no attempt was made to correct for source effects. In the case of major earthquakes, the finiteness effect could be significant, and is given by the following expressions (Press, Ben-Menahem, Toksöz, 1961)

$$U' = \frac{U}{1 + \frac{b}{2\Delta}\left(\frac{U}{V_f} - \cos\theta_0\right)} \approx U\left[1 - \frac{b}{2\Delta}\left(\frac{U}{V_f} - \cos\theta_0\right)\right] \tag{8}$$

where $U = d\omega/dk$, $U' = \Delta/t$ is the group velocity as measured from the seismogram, b = fault length, v_f = rupture velocity along the fault, and θ_0 = azimuthal angle. This could explain the apparent delay of the arrival of G_1 of the Assam earthquake by as much as 170 sec (arrival group velocity of 4.07 km/sec instead of the usual 4.35–4.40 km/sec).

Errors in Phase Velocity Measurements

It is important that the sources of error in phase velocity measurements are clarified before one can judge the accuracy of various measurements. Here we will briefly discuss the errors involved in Fourier analysis and direct measurement methods.

Fourier Analysis Method

In the phase velocity measurement with the Fourier analysis method using equation (2) only the phase term can be in error, provided the integer N is chosen correctly. The great circle distance may have the uncertainty of 10 km but this will only result in 0.025 per cent error in phase velocity since

$$\frac{\partial C}{C} = \frac{\partial \Delta}{\Delta}, \qquad \Delta \approx 40{,}000\,\text{km}. \tag{9}$$

The error in the phases is due to interference, noise, digitizing, and numerical inaccuracies. The latter two quantities are random errors and will show as scattering in the data. The extent of this error can be estimated from the behaviour of the phase spectra, and we find that in the frequency range of interest no measurable scattering occurs. Errors that may result from "window-shaping", and inadequate detrending are discussed by Gratsinsky (1962), but these can be avoided by proper care in the analysis. The interference (superimposition of two similar signals with a time delay, or two different signals with power in the same frequency range) is a serious problem in Fourier analysis. A typical indication of such interference is the presence of power-minima in the amplitude spectra accompanied by minima or turning points in the phase spectra (Pilant, 1962). The effect of the interference on the phases, and the error introduced is difficult to evaluate without knowing the true nature of the interference. In the case of two similar signals, one of which is delayed relative to the other by Δt, the error in the phase is

$$\Delta\phi = \frac{\alpha \sin \omega\Delta t}{\omega} \tag{10}$$

where α is the normalized amplitude of the delayed signal. The amplitude spectrum of the original pulse is modulated by the factor $(1 + \alpha \cos\omega\Delta t)$. The maxima and minima in the amplitude spectrum corresponds to $(1 + \alpha)$ and $(1 - \alpha)$ from which α can be found. The maximum phase error, $|\Delta\phi|_{\max} = \alpha\Delta t$, can be computed if Δt is known.

The error in the phase velocity due to phase inaccuracies is found by differentiating equation (2) with respect to $\delta\phi$.

$$\frac{\partial C}{C} = -\frac{CT}{\Delta}\,\partial(\delta\phi) \tag{11}$$

On the assumption that $\partial(\delta\phi) = 0.4$, the error in the phase velocity at a period of 100 seconds is less than 0.5 per cent. At longer periods, the error in the phases is much less than 0.4 circle. An important point should be restated here: errors in phase differences are due to interference and contamination introduced during propagation only, since the phase variations of the source are canceled when two successive passages of odd or even order waves are used.

Phase Velocities Measured Directly From the Seismogram

In the above discussion we pointed out that the Fourier analysis method of determining phase velocity is an exact method, and the only error results from the contamination of the phases. Time domain measurements, however, have theoretical as well as practical limitations. We will present here a brief discussion of these limitations.

The idea behind the method of direct measurement of phase velocities and initial phases from the seismogram is that one may associate a sine wave with each peak or trough of a dispersed wave train, which apart from the Airy phase, is shifted by $\pi/4$ with respect to the group arrival (Brune, Nafe, and Oliver, 1960; Nafe and Brune, 1960). Consider a surface wave of order n, which originates at an arbitrary source inside the isotropic, layered spherical earth. A component of the surface displacements at a station (θ, Δ) on the sphere at time t is given by

$$f_i(\theta, \Delta, t) = \frac{A(\theta)}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} S(\omega, \alpha_j) G(\omega, \beta_k) \begin{Bmatrix} W_n(\cos\eta) \\ \dfrac{\partial W_n(\cos\eta)}{\partial\eta} \end{Bmatrix} d\omega \tag{12}$$

where f_i is a component of the surface displacement, θ is the azimuth of the station with respect to the source, S is the source spectrum which depends on the parameters α_j, G is the medium response which depends on the parameters β_k, $A(\theta)$ is the azimuthal function of the source, and W_n is defined by

$$W_n(\cos\eta) = P_n + i\frac{2}{\pi}Q_n \approx \sqrt{\frac{2}{n\pi\sin\eta}}\,g(\eta)e^{i[\eta(n+1/2)-x-\frac{\pi}{4}]}. \tag{13}$$

The asymptotic form holds when

$$n + \frac{1}{2} = kR_0 \gg 1, \qquad \eta_0 < \eta < \pi - \eta_0, \qquad 0 < \eta_0 \leqq \frac{\pi}{6}$$

where

$$R_0\eta = \Delta, \qquad x = \tan^{-1}\frac{\cot\eta}{8n(1 - n/4)}$$

Inserting (13) in (12) and evaluating (12) by the method of stationary phase with the time t in the role of the large parameter, one finds:

$$\begin{aligned}[f_i(\theta,\Delta,t)]_m &= \frac{A(\theta)}{2\pi}\sqrt{\frac{2\pi}{t\left|\frac{d^2\omega}{dk^2}\right|}}\,\frac{N(\omega_0)}{\sqrt{R_0\sin\Delta_1}} \\ &\cdot\exp i\left[\omega_0 t - \Delta_m k + \frac{\pi}{4} \pm \frac{\pi}{4} + \frac{\pi}{2}(m-1) + \phi(k_0)\right]\{1 + i\epsilon_0\}\end{aligned} \tag{14}$$

$\pi/2\ (m - 1)$ is the polar phase shift, R_0 is the mean radius of the earth, Δ is the epicentral distance, k is the wave number, ω_0 is the value of ω at the point of stationary phase and is given by the intersection of the group velocity curve with the line $U = \frac{\Delta_m}{t}$, $\phi(k_0)$ is the initial-phase function, ϵ_0 is the magnitude of the first correction term. The transition from (12) to (14) is valid only if:

1. t is large enough to ensure a fully dispersed wave-train.
2. The vicinity of the polar region in the source coordinates is excluded.
3. Neither the source-function nor the layering function vary rapidly as a function of frequency.
4. $d^2\omega_0/dk_0{}^2 \neq 0$, that is, the frequency ω_0 is not too near an extremum of the group-velocity curve.
5. The 'Pekeris-condition' holds (Pekeris, 1948);

$$\epsilon_0 = \left|\frac{1}{\Delta_m}\left[-\frac{5}{24}\frac{(\dddot{k})^2}{(\ddot{k})^3} + \frac{\ddddot{k}}{8(\ddot{k})^2}\right]\right| \ll 1, \quad \dot{k} = \frac{dk}{d\omega}, \ddot{k} \neq 0 \tag{15}$$

6. Only one branch of the dispersion curve is considered, i.e. only one solution of $U = \Delta_m/t$ is taken.

The 'Pekeris-condition' introduces an additional phase of $\tan^{-1}\epsilon_0$, and the phase associated with each group-arrival on the record will be shifted by the amount $\tan^{-1} \epsilon_0$ with respect to the group. To evaluate this phase one can express the Pekeris-condition in terms of $\dot{U} = dU/dw$, and obtain for the leading term:

$$\tan^{-1} \epsilon_0 \approx \frac{1}{12} \frac{1}{\Delta_m} \left| \frac{dU}{d\omega} \right|. \tag{16}$$

The percentage error in the phase velocity due to this phase will be $\frac{1}{2\pi} \left(\frac{\lambda}{\Delta_m}\right)^2 \frac{T}{C} \left| \frac{dU}{dT} \right|$ which is negligible for periods in the region 100–700 seconds.

In addition to the theoretical restrictions listed above there are experimental errors due to the inability of reading the periods exactly, and the errors introduced in averaging where the periods are not matched exactly. Let us assume that two peaks, one with period T_1 and the other with T_2, are correlated. The average period is defined as $\bar{T} = (T_1 + T_2)/2$ and the phase velocity is computed for this average assuming that $C(\bar{T}) = \bar{C}(T)$. The error involved in this interpolation can be obtained using the results of Savarensky (1959). Leaving out the algebra involved in this process, the expression for phase velocity including the first correction term is:

$$\bar{C}(T) \approx C(\bar{T}) \left[1 - \frac{\bar{C}^2}{\bar{U}^2} \left(\frac{\bar{C}}{\bar{U}} - 1 \right) \left(\frac{T_2 - T_1}{T_2 + T_1} \right)^2 \right]. \tag{17}$$

The second term in the brackets is the error resulting from taking the arithmetic mean to determine the average periods.

The error that results from inaccuracies in reading the time and periods is more serious than the other sources of error. When the true period T_0 is determined erroneously to be T_1, the computed phase velocity is taken to be $\bar{C}(T_1)$ rather than $C(T_0)$. $\bar{C}(T_1)$, however, differs from the true phase velocity at period T_1, $C(T_1)$, because of the propagation time t in the expression for phase velocity. In determining $\bar{C}(T_1)$, $t = t_0$ was used instead of $t = t_1$. The first order error in reading the period can be written as

$$\frac{\partial C}{C} = \frac{\frac{\partial t}{\partial T} dT}{t - T\left(N - \frac{1}{2}\right)}. \tag{18}$$

$(\partial t/\partial T)dT$ is not the error in reading the time t inaccurately, but is the error caused by assigning a wrong propagation time to a given period. The magnitude of this error depends on the slope of the dispersion curve.

CONCLUSIONS

Phase velocities of mantle Love and Rayleigh waves can be measured accurately over a wide frequency range from the phase spectra of repeated passages of the same wave train. It is very important, however, that the pulse is free of interference and contamination; otherwise, undetermined errors will be introduced in the phases, and hence in the phase velocities. If properly used, the Fourier analysis method is especially valuable for application to mantle Love waves where the group velocity is nearly constant over a wide range of frequencies, and the wave retains a pulse-like shape.

The phase velocities of Love waves over different great circle paths are in good agreement where the paths are close. Where the paths are quite different, there are consistent variations in phase velocities, which are more pronounced at shorter periods. The phase velocities are higher over paths which are almost completely oceanic (New Guinea) than those over the paths that cross the continents. This shows that the upper mantle velocities vary laterally, and may be different under the oceans from those under the continents. Such a variation has also been observed from velocities of Rayleigh waves (Dorman, Ewing, and Oliver, 1960; Aki and Press, 1961; Kuo, Brune, and Major, 1962). We are, at the present, fitting structures to the observed phase velocities to determine the extent of variations over different paths.

The group velocity curve of Love waves has a wide plateau between the periods of 100 and 320 seconds with a shallow minimum around $T = 290$ seconds. For the predominantly oceanic paths the group velocities are higher than those for the other paths.

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SEISMOLOGICAL LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA
DIVISION OF THE GEOLOGICAL SCIENCES
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